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In blessed memory of Professor Lev Grigoriyevich Kachurin, research advisor

ON THE THEORY OF FORMATION OF A SANDWICH STRUCTURE OF A SPHERICAL HAILSTONE

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The paper analyzes the motion of the crystallization front under laminar and turbulent conditions of film motion on a hailstone surface. It is shown that the transition from one structure of ice to another is determined by both the equilibrium critical thickness of the film and the generalized Maclean parameter. A relation between them has been established. An expression for Kachurin's critical water content q_K defining the dry–wet transition of the hailstone growth has been obtained. It is shown that the Schumann–Ludlam critical water content q_{Sch-L} gives only that value of the critical content at which droplets settled on the hailstone freeze. If $q_{Sch-L} < q < q_K$, then a film is formed on the hailstone surface, but it is in an unstable state and, in the course of time, disappears, and the wet regime of the hailstone changes into the dry one. The hailstone will grow in the wet regime only at $q > q_K$.

Introduction. The mechanism of formation of a sandwich structure of ice growing in a supercool flow of aerosol was proposed in [1] and was first used in investigating the problem of airplane icing and then the problem of ice formation on sea vessels in a flow of splashes and supercool rain [1]. The point of the theory [1] is that on the surface of an object placed in a flow of supercool water aerosol a water film is formed. Under the action of the shear stress formed by the air flow it comes in motion. Depending on the film thickness and the air flow rate, its motion can be both laminar and turbulent. This in turn leads to two mechanisms of heat transfer: molecular and turbulent ones. In the first case, the film turns out to be unstable; it disappears, droplets crystallize without coalescing together and form a dull nonuniform structure of ice. In the second case, crystallization occurs under a steady thickness of the film and a transparent uniform structure of ice is formed thereby.

In [3–5], the mechanism proposed in [1] was considered on the assumption that the crystallization-front temperature T' was constant and equal to T_0 = 273 K. This made it possible to give an analytical solution to the problem of ice growth under the film. In the present paper, the results obtained by the author in [3–5] will be used to explain the sandwich structure of the spherical hailstone.

1. Turbulent Thermal Diffusivity. According to [6, 7], the turbulent thermal diffusivity k_t of the film on the hailstone surface can be given by the formula

$$k_t = b\,\frac{v^2\,(r - r')^2}{\nu}\,, \quad v = \sqrt{\frac{\tau_0}{\rho}}\,. \tag{1}$$

Let us introduce the mean thermal diffusivity $\bar{k}_t$ throughout the film thickness $h = r'' - r'$:

$$\bar{k}_t = \frac{1}{h}\int_{r'}^{r''} k_t dr = \frac{b}{3\nu}\,v^2 h^2\,. \tag{2}$$

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Between the dynamic velocity v and the characteristic motion of the film u_∞ there exists the following relation [8]:

$$v = \sqrt{\frac{c_f}{2}}\, u_\infty \,. \tag{3}$$

The drag coefficient c_f for the sphere is a function of the Reynolds number [8]. But for hailstones of radius $R \approx 0.003$–0.02 m (this corresponds to the range of Reynolds number Re = 10^3–10^5 [6, 8]) the drag coefficient is a constant c_f = 0.4 [9]. However, it should be remembered that the diversity of forms of natural hailstones and the roughness of their surface considerably change the value of c_f. For various forms of hailstones the values of c_f have been determined [9]. Below, in the theoretical studies we will assume that the hailstone is spherical and c_f is constant.

To find the relation between the flow rate of the film u_∞ and the rate of fall of the hailstone V, we assume that, according to [10], the turbulent friction remains constant and the friction of the crystallization-front surface τ_0 in the absence of droplet detachment from the hailstone surface is equal to the friction on the film surface F:

$$c_f \frac{\rho u_\infty^2}{2} = c_f \frac{\rho_{air} V^2}{2} \,. \tag{4}$$

In view of (4) for v we obtain the following expression:

$$v = \sqrt{\frac{\rho_{air} c_f}{2\rho}}\, V \,. \tag{5}$$

Substitution of (5) into (2) gives

$$\bar{k}_t = B' c_f V^2 h^2 \,, \tag{6}$$

where B' = 2.8 $(m^2/sec)^{-1}$.

As is known, the steady rate of fall of a hailstone V is calculated by the formula [9]

$$V = \sqrt{\frac{8}{3} \frac{\rho_{hsd}}{\rho_{air}} \frac{g}{c_f} R} \,. \tag{7}$$

Hence, substituting (7) into (6), for $\bar{k}_t$ we obtain

$$\bar{k}_t = B_1 R h^2 \,, \tag{8}$$

where $B_1 = 6.5 \cdot 10^4$ $m^{-1} \cdot sec^{-1}$. For example, for a hailstone of radius R = 0.01 and a film thickness $h = 10^{-3}$ m, $\bar{k}_t = 6.5 \cdot 10^{-4}$ m^2/sec, which is three orders of magnitude higher than the molecular heat conductivity coefficient of water $k = 1.3 \cdot 10^{-7}$ m^2/sec.

2. Ice Growth under the Film at Its Turbulent Flow. Once the expression for the turbulent heat-conductivity coefficient has been obtained, we can proceed to solving the problem of ice growth under the film. The heat-balance equation at the crystallization front is of the form [11]

$$\rho \left[L_{cr} - c\,(T_0 - T_\infty)\right] \frac{dr'}{dt} + c\rho \bar{k}_t \left.\frac{dT}{dr}\right|_{r=r'} = 0 \,. \tag{9}$$

In this case, according to [9], we neglect the heat outflow inside of the icy sphere, assuming the temperature of the whole icy sphere equal to T_0.

Assume that in a film of thickness $h = r'' - r'$ the stationary distribution of temperature T

$$\frac{dT}{dr} = -\frac{T_0 - T_1}{r'' - r'}\frac{r'r''}{r^2} \quad (10)$$

has come to a steady state.
Hence

$$\left.\frac{dT}{dr}\right|_{r=r'} \cong -\frac{T_0 - T_1}{h}\left(1 + \frac{h}{r'}\right). \quad (11)$$

From [11] at $h << r'$

$$\left.\frac{dT}{dr}\right|_{r=r'} = \left.\frac{dT}{dr}\right|_{r=r''} \cong -\frac{\Delta T}{h}, \quad \Delta T = T_0 - T_1 . \quad (12)$$

Find the mean value of the temperature gradient

$$\overline{\frac{dT}{dr}} = -\frac{T_0 - T_1}{r'' - r'} r'r'' \int_{r'}^{r''} \frac{dr}{r^2} = -\frac{\Delta T}{h} . \quad (13)$$

The equation of the hailstone growth due to the inflow of droplets is of the form [9]

$$\frac{dr''}{dt} = \frac{qEV}{4\rho} \equiv u . \quad (14)$$

In theoretical studies, we assume $E = 1$ [9].

Taking into account (8), (12), and (14), we write expression (9) in the form

$$\frac{dr'}{dt} = \frac{ut + R - r'}{\tau}, \quad (15)$$

$$\tau = \frac{L_{\mathrm{cr}} - c\,(T_0 - T_\infty)}{cB_1 R\Delta T} . \quad (16)$$

Write (15) in variables $h = R + ut - r'$ [5]:

$$\frac{dh}{dt} + \frac{h}{\tau} = u . \quad (17)$$

Find the steady-state thickness of the film at $dh/dt = 0$:

$$h_{\mathrm{s}} = u\tau = \frac{L_{\mathrm{cr}} - c\,(T_0 - T_\infty)}{4c\rho B'}\frac{qE}{c_{\mathrm{f}} V\Delta T} . \quad (18)$$

The solution of Eq. (17) is of the form [5]

$$h = h_0 \exp\left(-\frac{t}{\tau}\right) + h_{\mathrm{s}}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right). \quad (19)$$

From (19) it is seen that whatever the conditions $h_0 > h_s$ or $h_0 < h_s$ in the course of time the film thickness tends to h_s. Thus, in the turbulent regime of film motion, which is realized at a Reynolds number larger than the critical value $\mathrm{Re} \ge 1500$ [11], the hailstone grows in the so-called wet regime under the steady-state film thickness with the formation of a uniform structure of ice.

Introduce the notation $dr'/dt = w$. We represent the solution of Eq. (15) as

$$w = w_0 \exp\left(-\frac{t}{\tau}\right) + u\left(1 - \exp\left(-\frac{t}{\tau}\right)\right). \tag{20}$$

From (20) it is seen that in the course of time the rate of motion of the crystallization front tends to the rate of inflow of droplets.

3. Ice Growth under the Film at Its Laminar Flow. Consider the laminary regime of film motion on the hailstone surface. In this case, the heat-balance equation at the crystallization front will be of the form [11]

$$\rho\,[L_{\mathrm{cr}} - c\,(T_0 - T_\infty)]\,\frac{dr'}{dt} + \lambda\,\frac{dT}{dr}\bigg|_{r=r'} = 0\,, \tag{21}$$

where $\lambda = c\rho k$. In so doing, as in writing Eq. (9), we have neglected the heat outflow inside the icy sphere [9, 11]. Taking into account (12), we write (21) as

$$\frac{dr'}{dt} = \frac{k_0}{R + ut - r'}\,, \tag{22}$$

where

$$k_0 = \frac{\lambda \Delta T}{\rho\,[L_{\mathrm{cr}} - c\,(T_0 - T_\infty)]}\,. \tag{23}$$

Write (22) in variables h [5]:

$$\frac{dh}{dt} + \frac{k_0}{h} = u\,. \tag{24}$$

As is seen from (22) and (24), with increasing film thickness the rate of motion of the crystallization front decreases and the film growth rate tends to the rate of inflow of droplets. Find the equilibrium film thickness h_{eq} from the condition $dh/dt = 0$:

$$h_{\mathrm{eq}} = \frac{k_0}{u} = \frac{4c\rho k \Delta T}{qEV\,[L_{\mathrm{cr}} - c\,(T_0 - T_\infty)]}\,. \tag{25}$$

The solution of (24) is of the form

$$h - h_0 + h_{\mathrm{eq}} \ln\left|\frac{h - h_{\mathrm{eq}}}{h_0 - h_{\mathrm{eq}}}\right| = ut\,. \tag{26}$$

Taking into account that $h = R + ut - r'$, we write

$$|h - h_{\mathrm{eq}}| = |h_0 - h_{\mathrm{eq}}| \exp\left(\frac{r' - R + h_0}{h_{\mathrm{eq}}}\right). \tag{27}$$

If $h_0 > h_{eq}$, then $\left| h - h_{eq} \right| = h - h_{eq}$, and from (27) it follows that as the crystallization front advances, the film thickness increases, i.e., the rate of inflow of droplets is higher than the rate of motion of the crystallization front, which also follows immediately from formulas (22) and (24). But if $h_0 < h_{eq}$, then from (22), (24), and (27) it follows that with advancement of the crystallization front the film thickness will decrease. Immediately from (26), substituting $h = 0$, we can find the time during which the film with the initial thickness h_0 will disappear:

$$t_0 = \frac{h_0 \ln\left(\frac{h_{eq}}{h_{eq} - h_0}\right) - h_0}{u} . \tag{28}$$

From (28) it is seen that the disappearance time of the film largely depends on its initial thickness, the water content of the cloud, and the rate of fall of the hailstone.

Thus, the viscous equilibrium regime is unstable, since the slightest deviation of the film thickness h from h_{eq} will disturb the system from equilibrium: the film thickness will monotonically depart from the equilibrium thickness [1]. If at the initial time the film thickness is larger than the equilibrium thickness and the viscous regime of its motion (molecular mechanism of heat transfer) takes place, then with increasing film thickness the motion goes to the turbulent regime (turbulent mechanism of heat transfer).

4. Critical Film Thickness. Find the value of the critical film thickness at which the transition from the viscous regime of film motion to the turbulent one occurs. From (18) and (25) we obtain

$$h_s h_{eq} = k_0 \tau = \frac{\lambda}{c\rho B' c_f V^2} = \frac{\lambda}{\lambda_t} h^2 , \quad \lambda_t = c\rho \bar{k}_t . \tag{29}$$

From this we find the critical film thickness h_c from the condition $\lambda_t = \lambda$:

$$h_c = \sqrt{h_s h_{eq}} = \sqrt{k_0 \tau} = \sqrt{\frac{k}{B' c_f} \frac{1}{V}} . \tag{30}$$

Although both mechanisms of heat transfer exist concurrently [6], we assume, as is done in many problems in hydrodynamics [8], that at $h < h_c$ the chief mechanism of heat transfer is the molecular mechanism and at $h > h_c$ — the turbulent one.

As is seen from (30), the critical film thickness determining the transition from one structure of ice to another is not constant but varies with the rate of fall of the hailstone. From (17), (24), and (30) it follows that three different situations are possible:

(1) $h_{eq} < h_c < h_s$, with $(dh/dt)_c > 0$;

(2) $h_{eq} = h_c = h_s$, with $(dh/dt)_c = 0$;

(3) $h_{eq} > h_c > h_s$, with $(dh/dt)_c < 0$.

In the first case, at $h_{eq} < h_c < h_s$ the film motion is laminar and its thickness increases to the value of h_c at which the laminar motion of the film changes over to turbulent motion. Then the film thickness keeps increasing until the h_s value is reached. In the second case, the thickness of the film is constant and its motion is turbulent. In the third case, independent of the regime of film motion, its thickness decreases.

L. G. Kachurin [12] established the dependence of the structure of ice on the parameter λ_{eq}, which, according to formula (25), is a function of the water content and temperature of the cloud and the rate of flow. The parameter h_{eq} varied between $5 \cdot 10^{-4}$ m and $4 \cdot 10^{-2}$ m. It was established that there exists such a critical value of the equilibrium thickness h_{eq} (denote it by $h_{eq.c}$) at which it may be expected with a fair probability that $h > h_{eq.c}$. The value of $k_{eq.c}$ is determined from experiment and is equal to $1.3 \cdot 10^{-3}$ m [1, 12]. The notion of the critical equilibrium thickness is based on the probabilistic interpretation of experimental materials but does not follow at once from either Kachurin's theory [1] or the theory developed above. In our opinion, there exists an equilibrium thickness of the film determining its stability on the hailstone surface and assuming arbitrary values depending on the parameters defined by formula (25). The notion of $h_{eq.c}$ is still to be substantiated within the scope of the probability-statistical theory of hailstone growth.

Thus, if $h < h_{eq.c}$, then the molecular mechanism of heat conduction is observed, the film disappears, and droplets crystallize without coalescing together, forming a dull nonuniform structure of ice. But if $h > h_{eq.c}$, then the turbulent mechanism of heat conduction takes place, the film thickness, decreasing, tends to $h_{eq.c}$, and crystallization occurs under a constant film thickness with the formation of a transparent uniform structure of ice.

It also follows from (17) and (24) that to the stable state $dh/dt = 0$ there corresponds some critical value of the rate of inflow of droplets $u_c = k_0/h_{eq.c} = h_s/\tau$. Taking into account (23), we obtain

$$\frac{qEV}{4\rho} = \frac{k}{h_{eq.c}} \frac{c\Delta T}{[L_{cr} - c\,(T_0 - T_\infty)]} . \tag{31}$$

The transition from one structure of ice to another will also be determined by the generalized Maclean parameter [13]

$$\mathrm{M} = \frac{qEV}{\Delta T} \frac{4\lambda}{h_{eq.c}\,[L_{cr} - c\,(T_0 - T_\infty)]} . \tag{32}$$

Thus, both the Maclean parameter M and the equilibrium critical thickness of the film $h_{eq.c}$ characterize the transition from one structure of ice to another. And formula (32) establishes the relationship between the parameters M and $h_{eq.c}$: $\mathrm{M} \sim 1/h_{eq.c}$. In (32), at $T_0 - T_\infty = 10^{o}$C the ratio $L_{cr}/[c(T_0 - T_\infty) \approx 8$. Consequently, the term $c(T_0 - T_\infty)$ compared to L_{cr} can be neglected.

The Maclean parameters and $h_{eq.c}$ [1] are dimensional; therefore, it is convenient to introduce the dimensionless parameter

$$K = \frac{qE}{4\rho} \frac{L_{cr}}{c\Delta T} \frac{Vh_{eq.c}}{k} , \tag{33}$$

characterizing the transition from one structure of ice to another: at $K > 1$, a transparent uniform structure of ice is formed; at $K < 1$, there appears a dull nonuniform structure of ice. Formula (31) also permits obtaining the value of the critical water content q_K determining the transition from one structure of ice to another:

$$q_K = \frac{4\lambda\Delta T}{h_{eq.c}EV\,[L_{cr} - c\,(T_0 - T_\infty)]} . \tag{34}$$

For example, at $E = 1$ and $\Delta T = 1^{o}$C for a hailstone falling at a rate $V = 20$ m/sec the critical water content will be $q_K \approx 3 \cdot 10^{-4}$ kg/m^3.

5. Schumann–Ludlam Critical Water Content. The heat-balance equation on the film surface is of the form [14]

$$-4\pi R^2 \lambda \left.\frac{\partial T}{\partial r}\right|_{r=r''} = 2\pi R\lambda_{air}\,\mathrm{Nu}\,(T_1 - T_\infty) + 2\pi RD\,\mathrm{Sh}\,L\,(\rho_v - \rho_{v\infty}) . \tag{35}$$

The first and second terms on the right-hand side of (35) denote the quantities of heat leaving the film surface due to convective heat exchange and evaporation.

Taking into account the temperature distributions in the film (12) and the heat-balance equations at the crystallization front (21), we can write relation (35) in the form

$$4\pi R^2 \rho\,[L_{cr} - c\,(T_0 - T_\infty)]\,\frac{dr'}{dt} = 2\pi R\lambda_{air}\,\mathrm{Nu}\,(T_1 - T_\infty) + 2\pi RD\,\mathrm{Sh}\,L\,(\rho_v - \rho_{v\infty}) . \tag{36}$$

It is in this form that the heat-balance equation on the hailstone surface is written in the Schumann–Ludlam theory [9, 13]. As is known, in this theory Eq. (36) is obtained on the assumption that all droplets settled on the hailstone freeze, i.e., the film on the hailstone surface is absent. In this case, the heat released under crystallization is directly removed due to the evaporation and heat conduction. However, as shown above, Eq. (36) can also be obtained in the presence of film on the assumption that in the film on the hailstone surface a stationary temperature distribution takes place.

In Secs. 2 and 3, it is shown that the rate of motion of the crystallization front strongly depends on the regime of film motion on the hailstone surface. If such a film is absent and all droplets settled on the hailstone crystallize, then $dr'/dt = dr''/dt$ and the heat-balance equation at the crystallization front takes on the form

$$\pi R^2 VqE\,[L_{cr} - c\,(T_0 - T_\infty)] = 2\pi R\lambda_{air}\,\mathrm{Nu}\,(T_1 - T_\infty) + 2\pi RD\,\mathrm{Sh}\,L\,(\rho_v - \rho_{v\infty})\,. \tag{37}$$

From (37), in the Schumann–Ludlam theory one obtains the equation for the critical water content

$$q_{\mathrm{Sch-L}} = \frac{2\lambda_{air}\,\mathrm{Nu}\,(T_1 - T_\infty) + 2D\,\mathrm{Sh}\,L\,(\rho_v - \rho_{v\infty})}{RVE\,[L_c - c\,(T_0 - T_\infty)]}\,, \tag{38}$$

at which all water entrapped by the hailstone freezes. The quantity $q_{\mathrm{Sch-L}}$ gives only that value of the water content at which droplets that settled on the hailstone freeze. If $q_{\mathrm{Sch-L}} < q < q_K$, then on the hailstone surface a film is formed, but it is unstable. In time t_0 determined by formula (28), the film will disappear and the wet regime of the hailstone will give way to the dry regime. Only at a water content larger than Kachurin's water content ($q > q_K$) will the hailstone grow in the wet regime.

CONCLUSIONS

1. In the turbulent regime of film motion, which is realized when the Reynolds number is larger than the critical one, independent of the initial value, in the course of time the film thickness tends to the value of h_s. In so doing, the hailstone grows in the so-called wet regime under the steady film thickness, and a uniform structure of ice is formed thereby.

2. In the laminar regime of film motion, which is realized when the Reynolds number is smaller than the critical one, the film growth is determined by its equilibrium thickness. If $h_0 > h_{eq}$, then, as the crystallization front advances, the film thickness increases and at $h_0 < h_{eq}$ decreases. In the latter case, the hailstone grows in the wet regime with the formation of a dull nonuniform structure of ice.

3. For the hailstone to grow in the wet regime with the formation of a transparent uniform structure of ice, it is necessary that the water content of the cloud be higher than Kachurin's critical water content. If the water content of the cloud is higher than the Schumann–Ludlam critical water content but lower than Kachurin's critical water content, then on the hailstone surface a film is formed, but it is unstable and will disappear in the course of time, and, after this, the hailstone will grow in the dry regime with the formation of a dull nonuniform structure of ice.

NOTATION

q, water content of the cloud, kg/m^3; q_K, Kachurin's critical water content, kg/m^3; $q_{\mathrm{Sch-L}}$, Schumann–Ludlam critical water content, kg/m^3; T, variable temperature at an arbitrary point inside a liquid film; T', temperature of the film surface, K; T_0, freezing point of water, K; T_∞, air temperature in the cloud, K; λ and λ_t, molecular and turbulent heat-conductivity coefficients of water, J/(m·sec·K); λ_{air}, heat-conductivity coefficient of air, J/(m·sec·K); ν, coefficient of molecular viscosity of water, m^2/sec; D, diffusion coefficient of water vapor in air, m^2/sec; r', crystallization-front radius, m; r'', radius of the outer surface of the film, m; r, distance from the hailstone center to the internal point of the film, m; R, hailstone radius, m; h, film thickness, m; h_s, steady thickness of the film in its turbulent flow, m; h_{eq}, equilibrium thickness of the film, m; $h_{eq.c}$, critical value of equilibrium thickness, m; h_0, initial thickness of the film, m; h_c, critical thickness of the film at which the turbulent heat-conductivity coefficient is equal to the molecular heat-conductivity coefficient, m; v, dynamic velocity, m/sec; u_∞, characteristic rate of film motion at a large distance from the ice–water interface, m/sec; V, rate of fall of the hailstone, m/sec; $u \equiv dr''/dt$, rate of hailstone growth due to the inflow of droplets, m/sec; u_c, critical value of the rate of inflow of droplets, m/sec; w and w_0, rate and initial rate of motion of the crystallization front, m/sec; τ_0, friction stress at the crystallization front, N/r^2; F, friction stress on the film surface, N/r^2; ρ, ρ_{air}, and ρ_h, water, air, and hailstone density, kg/m^3; ρ_v, vapor density near the hailstone surface, kg/m^3; $\rho_{v\infty}$, vapor density in the environment at a long distance from the hailstone; kg/m^3; c_f, drag coefficient; b = 0.02, dimensionless empirical constant; B', constant, m^2/sec^{-1}; B_1, constant, m^{-1}, sec^{-1}; c, specific heat capacity of water, J/(kg·K); L_{cr}, specific heat of water crystallization, J/kg; L, specific heat of condensation (or evaporation), J/kg; E, entrapment coefficient; k_t, turbulent heat diffusivity of film, m^2/sec; k, molecular heat diffusivity of water, m^2/sec; t, time, sec; τ, characteristic time of motion of the crystallization front under turbulent flow of film, sec; M, Maclean parameter, kg/(m^2·sec·K); K, dimensionless parameter characterizing the transition from one structure of ice to another; g, free-fall acceleration, m^2/sec; Re, Reynolds number; Nu, Nusselt number; Sh, Sherwood number. Subscripts: t, turbulent; f, frontal; air, air; hs, hailstone; cr, crystallization, s, steady; eq, equilibrium; c, critical, v, vapor.

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